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Stress Fields and Strut-and-Tie Models as a Basic Tool for Design and Verification in Second Generation of Eurocode 2

Campos de tensiones y modelos de bielas y tirantes: herramientas fundamentales para el proyecto y comprobación de estructuras de hormigón en la segunda generación del Eurocódigo 2

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ABSTRACT

Within the frame of the revision of the Eurocode 2 for concrete structures, the section devoted to strut-and-tie design has been updated to enhance its applicability, its consistency with other sections and its ease-of-use. As a result, a number of changes have been introduced. Namely, the use of stress fields and their combination with classical strut-and-tie models has been incorporated. The changes in this section can be seen as an effort to provide a more comprehensive and general tool for designers, that can be transparently applied to any structural member with sufficient reinforcement for crack control. In this paper, the consistency between the strut-and-tie and the stress field methods is clarified as well as the fundamentals of the revision performed in Eurocode 2. The paper also elaborates how the code can be used for advanced analyses, considering in an explicit manner the compatibility of deformations to obtain refined estimates of the structural resistance..

KEYWORDS: Strut-and-tie, stress fields, limit analysis, shear, discontinuity regions, shell design.

RESUMEN

En el marco de la revisión del Eurocódigo 2 para estructuras de hormigón, se ha actualizado el capítulo dedicado al diseño mediante modelos de bielas y tirantes, mejorando su coherencia con otras secciones así como su facilidad de uso. Para ello, se han introducido una serie de cambios, como la consideración del método de campos de tensiones y su combinación con los modelos clásicos de bielas y tirantes. Estos cambios pueden considerarse como un esfuerzo para proporcionar una herramienta más completa y general. En este artículo, se clarifica la coherencia entre los métodos de bielas y tirantes y los campos de tensiones, así como los principios de la revisión efectuada para el Eurocódigo 2. También se explica cómo puede utilizarse dicha norma para realizar análisis avanzados, considerando de manera explícita la compatibilidad de deformaciones con el objetivo de obtener estimaciones precisas de la resistencia.

PALABRAS CLAVE: Bielas y tirantes, campos de tensiones, análisis límite, cortante, regiones de discontinuidad, diseño de lajas.

1. INTRODUCTION

Reinforced concrete as a structural material was introduced at the end of the 19th century through a number of patents [1-3]. Almost from the beginning, the engineers realised that traditional design methods rooted in linear elastic theory could not adequately be used to explore the full potential of the new composite material. A new approach was therefore needed and within

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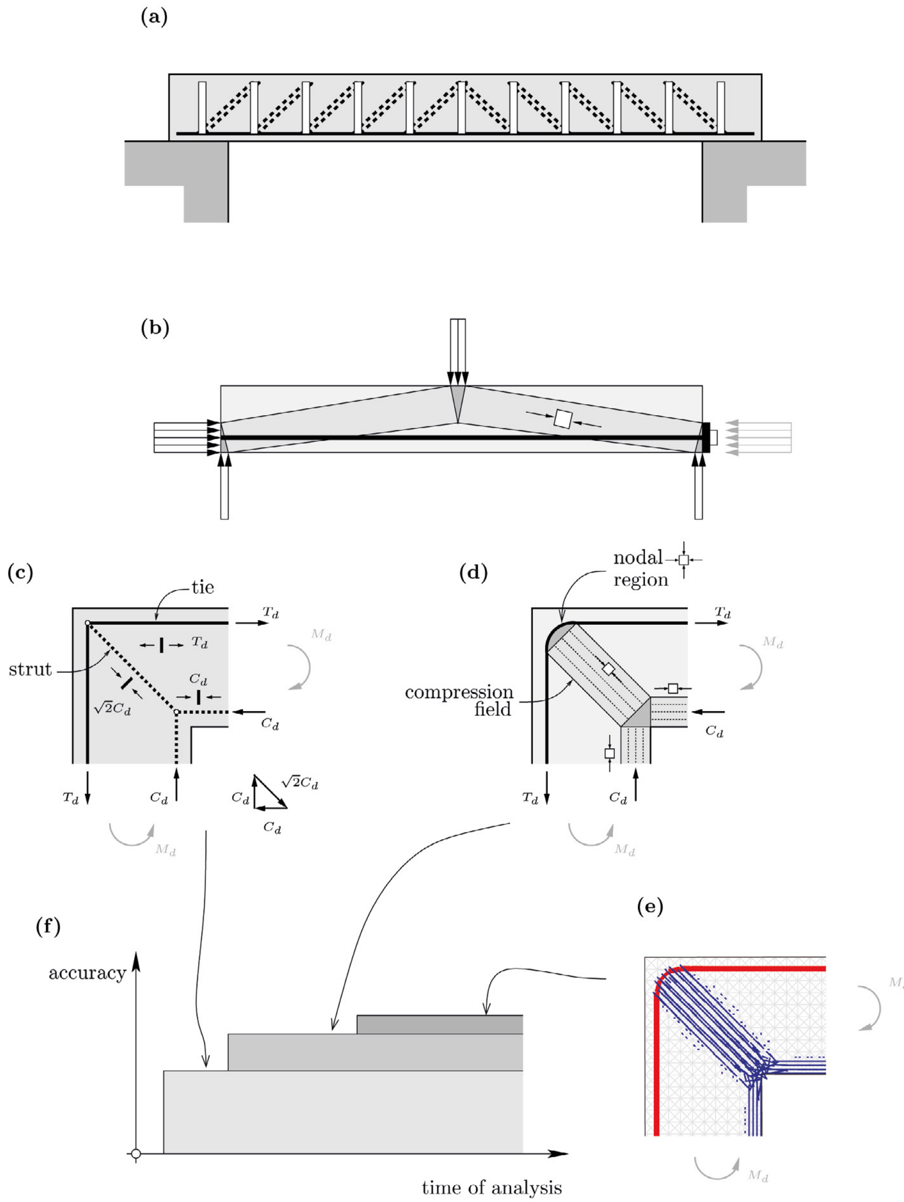

Figure 1. Strut-and-tie and stress fields: (a) truss model (adapted from Ritter [4]); (b) stress field (adapted from Drucker [9]); (c) strut-and-tie model; (d) corresponding rigid-plastic stress field; (e) elastic-plastic stress field; and (f) Levels-of-Approximation approach.

less than a decade, Ritter in 1899 [4] proposed in his pioneering work an engineering approach to characterize the load-carrying actions in cracked reinforced concrete beams. The approach included an idealization of the flow of forces within the cracked concrete element by means of an internal truss system (with concrete struts carrying compression forces and ties representing the reinforcement carrying the tensile forces, see Figure 1a). This concept provided a simple way to understand and to visualize how the element carries loads in the cracked state.

The approach by Ritter was adopted by the community of designers and was given the name *truss analogy* with reference to the widely used steel trusses at that time. The method was further developed by Mörsch [5] and later generalised beyond beam design by, amongst others, Leonhardt and co-workers in Stuttgart, (see for instance [6]). The approach was observed to be particularly efficient for design of the so-called *discontinuity regions*, where the Navier-Bernoulli assumption (i.e. plane sections remain plane) does not hold. Following this school, Schlaich and co-workers systematically developed criteria and guidance on how to arrange appropriate strut-and-tie systems (inspired by the stress trajectories in linear-elastic members and including non-prismatic struts). The work of Schlaich et al. [7-8] had a significant impact in practice and it became clear to the wider engineering community that the method in fact is grounded on the lower-bound theorem of limit analysis and, as such, could be used in a safe manner for design of new structures. During the course of development and generalization of the method, the term *strut-and-tie* was introduced (Figure 1c), representing the resultants of internal stresses and/or forces (the definition "truss analogy" was deliberately abandoned since in many cases, strut-and-tie models become labile trusses which depend on the load configuration).

While strut-and-tie modelling originated from the truss analogy and only in retrospective recognized as a lower bound method, the concept of stress field modelling for structural concrete was from its infancy directly based on application the lower bound theorem of limit analysis and rigid-plastic theory. The first stress field solutions, and corresponding failure mechanisms, were developed by D. Drucker in 1961 [9] who considered beams without shear reinforcement, see Figure 1b. During the 1960's - 1980's the potential of stress field modelling was utilised extensively, mainly by Thürlimann and co-workers in Zürich and by Nielsen and co-workers in Copenhagen, to address multiple relevant situations in structural concrete (for an overview, see e.g. [10-13]). The method allowed the engineers to take an active role in the design process, by choosing the arrangement of the reinforcement and consequently defining the actual flow of the internal forces.

The classical stress field approach has in recent years experienced a revitalization, where e.g. efficient numerical optimization algorithms have been utilised to solve large-scale problems [14-15]. The approach has furthermore been extended to allow implementation of elastic-plastic [13] as well as nonlinear constitutive laws [16-18], in order to explicitly account for (local) compatibility of deformations (see Figure 1e).

The different origin of classical strut-and-tie modelling and of stress field modelling means that engineers in the past (and perhaps still today due to code formulations) would arrive at different structural layout depending on which of the two concepts/schools they are most acquainted to. Designs based on strut-and-tie models tend to have a discrete nature with extensive use of concentrated reinforcement and with large zones of concrete assumed to be stress-free. On the other hand, designs based on stress fields tend to have a more continuous nature involving potentially smeared stress fields and mesh reinforcement. The different origin of the two methods is also reflected in the way they were implemented during the 1990's into the current version of Eurocode 2 (EN1992-1-1:2004 [19]). In this code, the provisions related to strut-and-tie modelling are very much inspired by the approach formulated by Schlaich et al. [8] and they have almost no connections/references to the plasticity-based methods also implemented in the code for shear design of members with shear reinforcement and for reinforcement design of membrane elements (see e.g. above-mentioned references).

It is, however, important to note that a stress field can always be represented by means of the resultants of stresses (in compression and tension) which in turns leads to a strut-and-tie model (see Figure 1c-d). Both approaches can therefore be seen as complementary tools in the design process [13,20], where the strut-and-tie models are particularly suitable to determine the total amount of reinforcement required in a certain area, while the stress fields allow for a detailed check of the compression fields and nodal regions. The complementary nature of the two methods suits perfectly the so-called Levels-of-Approximation approach, by which successive refinements of the design can be obtained (Figure 1f, [20,21]).

The complementary nature of the two methods will become clearer in the next generation of Eurocode 2 (EN1992-1-1:2004), whose current stable draft is the FprEN1992-1-1:2022 [22]. In this revised version of the code, the provisions concerning strut-and-tie design have been extensively updated and expanded into a new section named "Design with strut-and-tie models and stress fields". The intention of this new section is to provide the concepts of strut-and-tie models and stress fields in a consistent manner within the plasticity-based framework for ULS design. In this paper, the reasons for improvement of the EN1992-1-1:2004 clauses as well as its implementation and background are presented and discussed. The paper highlights the benefits of the changes, shows a practical design example and discusses on the overall consistency with other parts of FprEN1992-1-1:2022.

2. STRUT-AND-TIE AND STRESS FIELDS: DESIGN AND ASSESSMENT OF CONCRETE STRUCTURES AND REASONS FOR CHANGE IN FPREN1992-1-1

Stress fields and strut-and-tie models are used both for design of new structures as well as for the assessment of existing ones. In the past, codes have been fundamentally oriented towards the design of new structures. However, it can be expected that new generations of codes will meet the demand to have rules and methods explicitly dealing with assessment of existing structures. For instance, FprEN1992-1-1:2022 has a dedicated Annex (Annex I) for the assessment of existing structures, allowing for the use of advanced methods to determine more accurate estimates of the load carrying capacity of members, which e.g. do not fulfil the detailing rules related to new design.

In the frame of limit analysis, design and assessment can be performed following specific considerations taking advantage of the lower- and upper bound theorems of limit analysis [23]. For design, it is convenient to work with different lower bound models (Figure 2a) to decide on the manner to carry the loads within the structure. This gives enhanced freedom to tailor the geometry of the structure and to arrange the reinforcement in the most suitable manner. For assessment of existing structures, the context is different. Here, the geometry and reinforcement arrangement are given and the primary objective is typically to determine the maximum load that can be carried by the structure in order to decide whether the structure needs strengthening. To that aim, upper bounds of the load-carrying capacity based on considerations of different collapse mechanisms

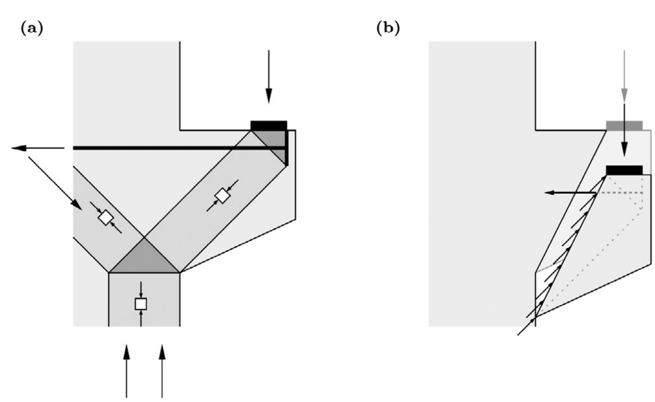

Figure 2. Example of lower- and upper-bound solutions: (a) lower bound (stress field); and (b) upper-bound (mechanism).

(Figure 2b) are particularly useful in the initial phase, as they are easier to establish - especially when dealing with complex geometries and reinforcement arrangements. For more refined estimates of the resistance, supplementary stress fields may be established to determine the gap between the upper- and the lower bound estimates. This strategy allows approaching the exact solution according to limit analysis (when an upper- and a lower bound solution meet [23]).

Exact solutions can be established in simple cases by following hand-made procedures [23]. However, numerical approaches can be needed in complex cases. Such approaches are already available in practice, e.g. efficient optimization procedures to establish the optimum rigid-plastic solution (see e.g. [14-15]) or finite element models based on elastic-plastic material behaviour to determine the load-carrying capacity and the displacement field at collapse (see e.g. [13,24]). The elastic-plastic approach has the advantage of providing equilibrium solutions that fulfil the yield conditions with proper consideration of the strength reduction factors (refer to next section) and at the same time ensuring compatibility of deformations. This eventually leads to a stress field with a corresponding licit failure mechanism and can therefore be interpreted as exact solutions within the frame of limit analysis. Such numerical approaches are state-of-the-art and can be safely used for design [25,26], although they require consideration of more advanced concepts than simple strut-and-tie provisions which can be found in EN1992-1-1:2004.

The necessity for a code addressing the challenges and needs of the structural engineers for the next decades suggested to evolve the provisions of EN1992-1-1:2004 in a series of topics:

- Keeping simplified procedures providing an enhanced freedom for design (lower-bounds), but allowing for advanced procedures (accounting for compatibility of deformations and consistent with the lower bound methods) to be used in e.g. design optimization and assessment
- Generalizing the strut-and-tie method and considering its combined use with stress fields to verify in a more transparent manner the compression fields and nodal regions
- Providing a more consistent integration of the provisions for design with strut-and-tie and stress field models with other sections of the code. This includes notably the sections on shear, bending and torsion design of linear members with web reinforcement and the Annex on design of membrane, slabs and shell elements

As it can be noted, such changes will enlarge the field of application of the section (with a special significance of assessment). The changes also allow for a progressive refinement of the analysis [21], starting with simple (hand-made) approaches covering most design situations and ending with refined (strain-based) approaches to obtain more accurate estimates, if required.

3. STRESS FIELDS FUNDAMENTALS

The provisions of the FprEN1992-1-1:2022 consider, according to the stress field method, that the external actions are equilibrated by a set of compression fields and tension ties converging at nodal regions. The compression fields, ties and nodal regions can be of concentrated or smeared nature. For instance, in Figure 3a smeared compression fields and ties are considered, while in Figure 3c, they are concentrated elements. On that basis, the corresponding strut-and-tie models can be

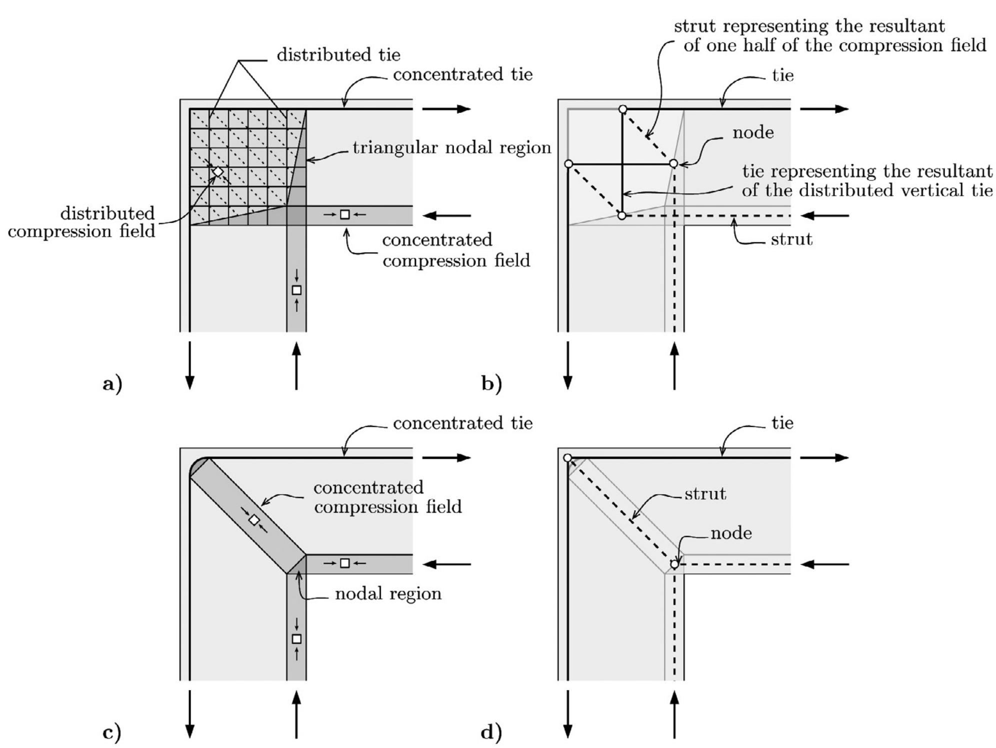

Figure 3. Strut-and-tie and stress fields: (a) stress field considering distributed compression fields and ties; (b) corresponding strut-and-tie model; (c) stress field considering concentrated compression fields and ties; and (d) corresponding strut-and-tie model.

determined, by arranging struts and ties as connectors between nodes of the stress field model (see Figure 3b,d). It can be noted that when a compression field in a panel transfers the forces between its edges (see Figure 3a), the resulting stress field can be designed following a stringer-panel approach [27,28]. In that case, the distributed compression fields result from membrane conditions and the forces at the edges are equilibrated by stringers (concentrated struts and ties).

In order to safely apply approaches based on limit analysis, the structure/member should have sufficient deformation capacity so that redistributions of stresses can occur. In structural concrete, such condition requires normally the member to be provided with a minimum amount of reinforcement (also covering other aspects as the crack localization at serviceability limit state, robustness or to avoid performing other detailed checks) and that the reinforcement has large deformation capacities (normally Ductility Class B or C according to FprEN1992-1-1:2022). These minimum requirements are normally sufficient to prevent crack localization (leading to smeared strains in the member) as well as brittle reinforcement rupture and thereby ensure safe application of the stress field method [20]. It shall be stated that more advanced models can be used to account for reinforcement with limited deformation capacity [20], but the FprEN1992-1-1:2022 does not explicitly suggest methods for so doing. In the following, the methods proposed by FprEN1992-1-1:2022 to verify the resistance of the compression fields and nodal regions will be introduced.

3.1. Compression fields

The verification of compression fields is performed in a direct manner on the basis of the acting stresses and the resistance of the material accounting for its state of strains (as a direct condition for the solution to be considered a lower-bound). This is formulated as follows:

$$\sigma_{cd} \leq \nu f_{cd} \tag{1}$$

where σ_{cd} is the stress at the location to be verified, f_{cd} is the design value of the compression strength of concrete and ν is the strength reduction factor to account for the detrimental influence of transverse strains (the so-called efficiency factor). It shall be noted that in the provisions of EN1992-1-1:2004, the ν-factor accounts for strength reduction due to transverse strains as well as concrete brittleness. However, in FprEN1992-1-1:2022, the two effects are separated into two factors, with ν solely reflecting the effect of transverse strain (see below) while the effect of brittleness is incorporated into the formulation of f_{cd}:

$$f_{cd} = \frac{f_{ck}}{\gamma_C} \eta_{fc} \tag{2}$$

where f_{ck} and γ_C are, respectively, the characteristic compressive cylinder strength and the partial safety factor. The coefficient η_{fc} is a strength reduction factor, which takes into account the post-peak strain-softening behaviour of concrete, when subjected to uniaxial compression (Figure 4a). This coefficient is in FprEN1992-1-1:2022 given as:

$$\eta_{fc} = \left(\frac{40}{f_{ck}}\right)^{1/3} \leq 1 \tag{3}$$

The formulation of FprEN1992-1-1:2022 adopts the format originally proposed by Muttoni in 1991 [29] and later adopted by Model Code 2010 [30], however with a slightly different reference value (40 MPa instead of 30 MPa). This change is introduced to account for a uniform reliability level for different concrete strengths [31] while keeping a constant value of γ_C.

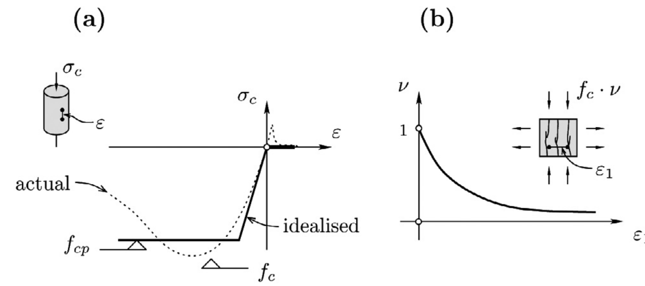

Figure 4. Concrete response: (a) idealised and actual uniaxial response of concrete; and (b) compression softening effect for an element in longitudinal compression and transverse tension.

The influence of transverse cracking on the effective compression strength of concrete is as mentioned above covered by the ν-factor (schematically shown in Figure 4b). It was Robinson and Demorieux in 1968 [32] who first documented this phenomenon in tests with membrane elements subjected to biaxial tension-compression. Several researchers have later on studied the phenomenon by means of panel shear tests (as those performed by Vecchio and Collins (1986 [33])), and on that basis suggested constitutive models for concrete which account for the influence of transverse cracking (a detailed state-of-the-art on this topic can be consulted in [20]). It is common in these models, that the first principal tensile strain, ε_1, is adopted as a measure of the level of transverse cracking (thereby assuming the members being sufficiently reinforced to avoid crack localization).

The general approach to account for the influence of strains on the concrete strength is described in the code (FprEN1992-1-1:2022) by means of the following expression:

$$\nu = \frac{1}{1+110\,\varepsilon_1} \tag{4}$$

where ε_1, as mentioned above, refers to the principal tensile strain (only tensile strains considered, see Figure 4b). This expression is similar to others in the literature and has been developed to suitably fit the results of panels tested under shear and normal forces (refer to Background document to Annex G of FprEN1992-1-1:2022 [34]). It shall be noted that by separating the effect of transverse strains and that of concrete brittleness, it is possible to adopt the same expressions for ν and η_{fc} in all of the provisions in FprEN1992-1-1:2022 dealing with strut and tie and stress field modelling. This is an advantage compared to EN1992-1-1:2004. It shall be noted that the evaluation of the ν-factor (being dependent on ε_1) has to be performed in an indirect manner when rigid-plastic stress fields are used for design purposes. A direct evaluation of ε_1 (and thereby of the ν-factor) will require methods which are able to determine stress fields that satisfy the compatibility conditions [13], as elastic-plastic methods.

For most cases in practice, however, a direct calculation of ε_1 is unnecessary for design. In fact, it is normally sufficient to assume that the main reinforcement reaches yielding and, on this basis, use the compatibility condition to derive a value for the ν factor. For instance, for beams in bending (Figure 5a) or when a gradient of strains can be assumed through the borders of a panel, the compatibility condition will lead to the following expression (refer to Mohr's circle in Figure 5b):

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x + 0.001)\cot^2\theta_{cs} \tag{5}$$

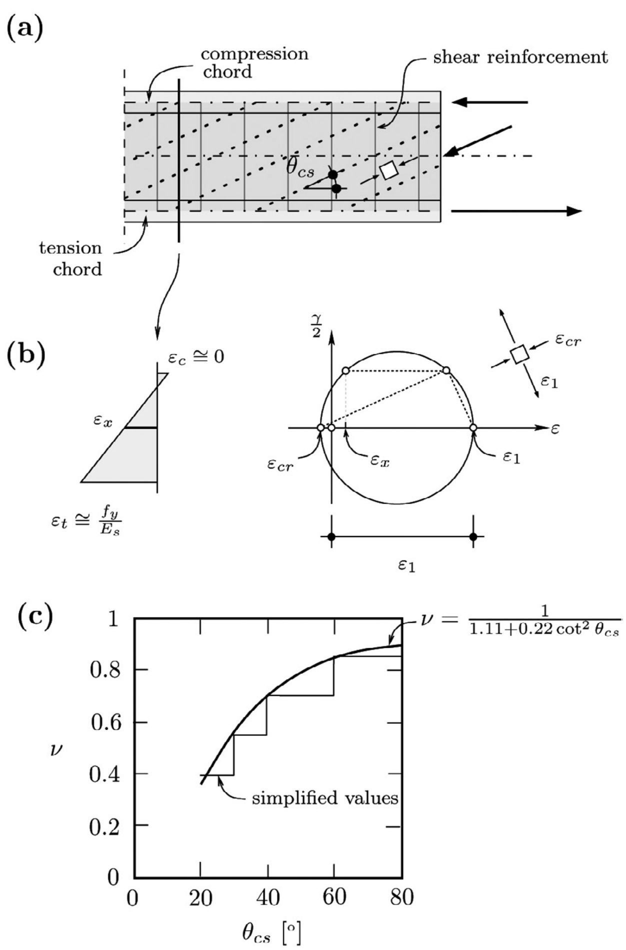

Figure 5. Verification of compression field in webs: (a) variable-angle truss model and direction of compression field; (b) Mohr's circle of strains (at mid-height); and (c) comparison of refined expression for calculation of ν and simplified values.

Where the parameters on which the maximum tensile strain (ε_1) depends are known:

- concrete is assumed at crushing to have a strain equal to approximately -0.1% (for an elastic-plastic response),
- θ_{cs} refers to the angle of the compression field with respect to the x-axis, and
- ε_x refers to the average strain between the top and bottom chords (that, for beams in bending, can be estimated by neglecting the strain of the compressive chord as $\varepsilon_x = \dfrac{f_y}{2E_s} \cong 0.001$.

And thus, the general expression results in this case:

$$\nu = \frac{1}{1.11 + 0.22\cot^2\theta_{cs}} \tag{6}$$

This expression, which is consistent to the one proposed for shear design of beams in FprEN1992-1-1:2022, allows for a detailed calculation of the efficiency factor under the previous assumptions (reinforcement yielding in a panel subjected to a gradient of strains), which covers a large number of cases. It can be further simplified to constant values for convenience, as for instance (figure 5c):

- $20° \leq \theta_{cs} < 30°$ $\nu = 0.40$
- $30° \leq \theta_{cs} < 40°$ $\nu = 0.55$
- $40° \leq \theta_{cs} < 60°$ $\nu = 0.70$
- $60° \leq \theta_{cs} < 90°$ $\nu = 0.85$

For other cases, analogous expressions can be derived depending on the strain conditions of the element. It is also important to note that angles between the strut and the tension ties lower than 20° are not allowed. This ensures that the values of the efficiency factors comply with those stated by the code. However, lower values could be derived if a refined analysis is performed accounting for compatibility of strains.

3.2. *Ties*

The ties, ensuring the transfer of tensile forces between the loads and/or nodal regions, can be designed or verified by respecting the condition of plasticity (where the acting forces Ftd shall be lower or equal to the resistance of the ties F_{Rd}):

$$F_{td} \leq F_{Rd} = A_s\, f_{yd} + A_p\, f_{pd} \tag{7}$$

Where A_s and f_{yd} refer to the area and yield strength of the reinforcing steel and A_p and f_{pd} to the area and yield strength of the prestressing steel (to be reduced accordingly if the prestressing force is considered as an external action).

3.3. *Nodal regions*

With respect to the nodal regions, they refer to the zones where the forces are transferred amongst the converging struts and ties. Depending on their configuration (where "C" stands for compression and "T" for tension), they can be classified as:

- CCC nodes: where only compression fields converge to the nodal region
- CCT nodes: in presence of one tie
- CTT nodes: in presence of two ties and one strut
- TTT nodes: with only converging ties

CCC nodes are the most favourable case. They can consist of three or more converging struts, see Figure 6. Such nodal regions are not typically governing for design. Provided that all struts carry the same level of stress, the nodal region can be in a hydrostatic in-plane state of stresses, directly fulfilling the resistance condition (ν=1). In case, where the stresses of the converging struts are not identical, a local spreading of the struts can be assumed (ensured by the minimum reinforcement of the member) or non-hydrostatic nodal regions can be considered. Also, connecting CCC triangular nodal regions by uniaxial compression fields is a suitable strategy in many cases of complex nodal geometry [35].

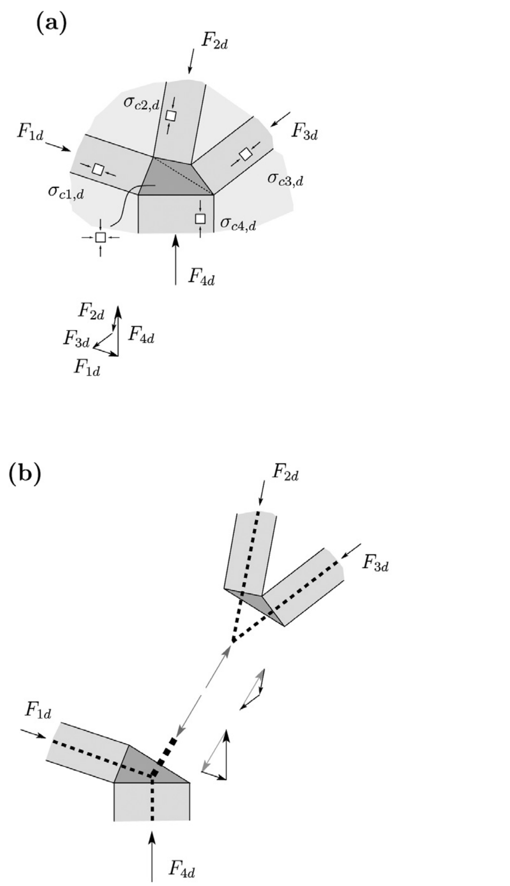

Figure 6. CCC node with four converging struts: (a) detail of the nodal region; and (b) analysis as two CCC nodes of three converging struts.

With respect to CCT nodes and CTT nodes, they can be of smeared or concentrated nature. For concentrated nodes, anchorage of the reinforcement can be provided outside of the nodal region, ensuring in-plane hydrostatic conditions (ν=1 inside the nodal region, see Figure 7a). In this case, the nodal regions are thus governed by the resistance of the converging struts (depending on their angle with the ties). For smeared configurations, the arrangement of the region shall satisfy the anchorage length of the bars, which is usually governing and enhanced by the presence of transverse pressure (see Figure 7c with reduced anchorage length). Intermediate cases of partial anchorage inside of a nodal region are also accepted (Figure 7b) in FprEN1992-1-1:2022, where the ν factor for the node is adopted in a simplified manner as a linear interpolation between the extreme cases (full anchorage outside of the nodal region and full anchorage within the nodal region).

Finally, concerning TTT nodes, its use is discouraged as the evaluation of the ν factor is subjected to uncertainties. In case a TTT node is identified, it is advised to modify the layout of the strut-and-tie or stress field model to avoid it (or to prestress one direction).

4.
REFINED ANALYSES

As previously explained, the approach of stress fields, and particularly its integration within FprEN1992-1-1:2022 allows for an easy implementation by means of numerical analyses. This is explicitly acknowledged in Annex I of FprEN1992-1-1:2022 and can for instance be done following the Elastic-Plastic Stress Field (EPSF) method. Within this approach, the yield conditions of the materials are introduced following an elastic phase [13]. The solution can be obtained in an automated manner based on a classic stiffness-based approach (implemented for instance by means of the Finite Element Method), where a displacement field is calculated fulfilling equilibrium, compatibility of deformations and material constitutive laws (considering the plastic response of concrete).

For the reinforcement, simple link elements can be considered with an elastic-plastic response (with or without strain-hardening), see Figure 8a. This ensures compatibility of deformations as well as respecting the yield conditions of the material. For concrete in uniaxial as well as biaxial compression, an elastic-plastic law can also be adopted. Furthermore, by neglecting the tensile strength of concrete, it is possible to work with a simple quadratic yield surface for plane stress conditions (Figure 8d), corresponding to a Mohr-Coulomb yield condition with a zero-tension cut-off. For plane stress conditions, usual and safe assumption for application of the stress field method, the concrete is subjected to a biaxial state of stresses. Thus, the yield criterion of concrete considers a Mohr-Coulomb yield condition with a tension cut-off (Figure 8d). The plastic strength is reduced consistently with the efficiency factor ν (depending on the local state of strains). The stress state is deter-

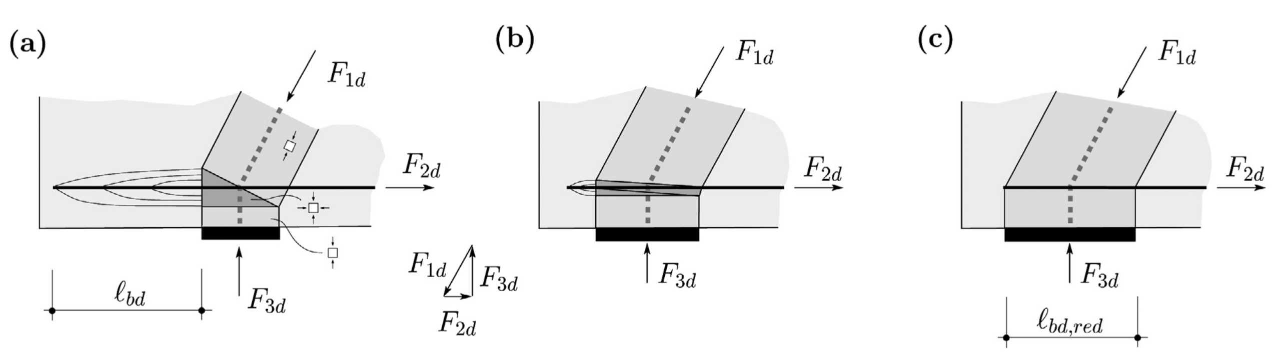

Figure 7. CCT node: (a) concentrated; (b) partly anchored within the nodal region; and (c) fully anchored within the nodal region.

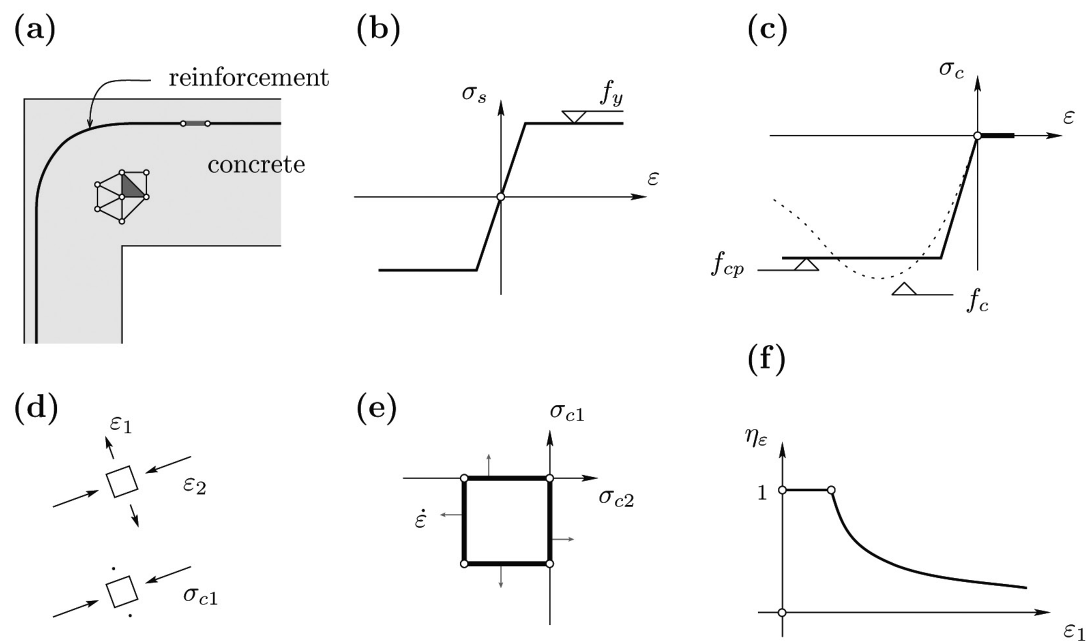

Figure 8. Finite Element Method implementation of Elastic-Plastic Stress Fields: (a) model; (b) constitutive law for steel; (c) strain and stress principal directions; and (d) yield criterion for concrete.

mined considering that the principal directions of the stress tensor and of the strain tensor are coincident, consistently with Nadai [36], Hencky [37] and the tension field model of Wagner [38]. As discussed by Prager [39] and in *fib* [20], for advanced states of deformation (development of a kinematically compatible mechanism), the deformation of the materials in the plastic regions are very large, ensuring convergence to a plastic solution where the stress tensor is considered parallel to the tensor of increment of plastic strains (considering same values for the efficiency factors).

The advantage of this approach is that the compatibility of deformations is respected locally, and thus refined estimates of the strain state and the corresponding ν factor can be determined. Also, failure occurs when a kinematically-admissible mechanism develops, ensuring the conditions for an exact solution according to limit analysis (for comparable values of efficiency factors).

The EPSF allows thus for refined estimates of the strength. It considers that the element and materials have sufficient deformation capacity to develop their yield plateau, allowing for potentially large stress redistributions (bounded by the resistance of the materials and namely by the weakening of concrete due to transverse cracking). As discussed above, in order to ensure sufficient deformation capacity of the materials, a minimum amount of reinforcement shall be provided in both directions, avoiding strain localization (associated to brittle failures). This minimum reinforcement shall at least comply with the amount required for elements in shear, and can locally need to respect other conditions depending on the response of the member (such as minimum reinforcement for bending or tension). In addition, the reinforcement shall have sufficient deformation capacity (typically class B or C according to EN1992-1-1:2004). Otherwise, performing a control of its deformation capacity accounting for the effect of bond on the rupture strain of the reinforcement is required [17,18].

5.
EXAMPLE OF APPLICATION

As an example of application, Figure 9a shows the support of a girder built for a project in Lausanne (Switzerland). It refers to a courtyard of a school under refurbishment, where reinforced concrete girders with a slenderness of approximately 13.6 (= 12.1/0.89) and 400 mm of width were arranged. The critical detail locates at the left support shown in Figure 9b, where questions arise on how shall such reinforcement be detailed. A rough analysis based on equilibrium considerations (thrust-line analysis), see Figure 9c, shows that, as expected, compression forces develop on the top face, while tension forces develop on the bottom face. However, the thrust line of the compression develops outside of the concrete element, and the thrust line of the tension will require to be deviated to remain within the member.

On that basis, a preliminary strut-and-tie model can be established, see Figure 9d. The model allows locating the main strut and ties and ensuring equilibrium. With this model, the benefits of arranging an inclined reinforcement can be easily acknowledged. Also, it can be noted that the region at the right of the bent of the flexural bar behaves in a similar manner as the end-region of a beam, with a conventional strut-and-tie arrangement. The analysis of the nodal regions shows a CCC node (node B in Figure 9d), a CCT node (node A in Figure 9d) and a CTT node (node C in Figure 9d). It can be noted that for the nodes CCT and CTT, the angle between the strut and the ties does not respect (at nodes A and C) the minimum angle θ_{cs} = 20° recommended by the FprEN1992-1-1:2022.

Grounded on these observations, some refinements can be introduced in the strut-and-tie model, Figure 9e, by providing spreading of the struts. Such spreading allows fulfilling the requirements in terms of minimum angles between struts and ties, and needs the arrangement of additional reinforcement in the form of horizontal and vertical bars (stirrups or pins, see Figure 9f). With respect to the region at the right of the bent of the bar, the fan region (with a steeper angle of the resulting strut) and the constant-angle compression field region (with a flatter angle of the resulting strut) can be designed following the standard procedure for shear in members with transverse reinforcement.

Finally, detailed checks can be performed on the basis of stress fields at the critical regions (nodal regions A and B), ensuring that sufficient space is available for development of the struts and nodal regions. To that aim, a constant and safe value of the efficiency factor is adopted (ν = 0.55 accounting for the angles of the struts and ties), allowing to analyse all nodes under plane-stress hydrostatic conditions, see Figure 9f. The results show that this aspect is not critical. Also, detailing of the reinforcement can be consistently established, in terms of type of reinforcement and anchorage lengths.

For a final optimisation, or in case the performance of the detailed needed to be assessed, a refined EPSF analysis could also be performed. The results are shown in Figure 10 for two cases. The first considers only inclined reinforcement and stirrups at the right of the bend (Figures 10a-d), corresponding to the reinforcement layout of Figure 9d. The latter considers also an additional horizontal and vertical reinforcement in the discontinuity region (Figures 10e-h), corresponding to the reinforcement layout of Figure 9f. In all cases, the load was applied by means of a stiff plate, distributing it into the concrete surface.

When only inclined reinforcement is provided in the discontinuity region (Figures 10a-d), a similar response to that of Figure 9d results, with an inclined compression field devel-

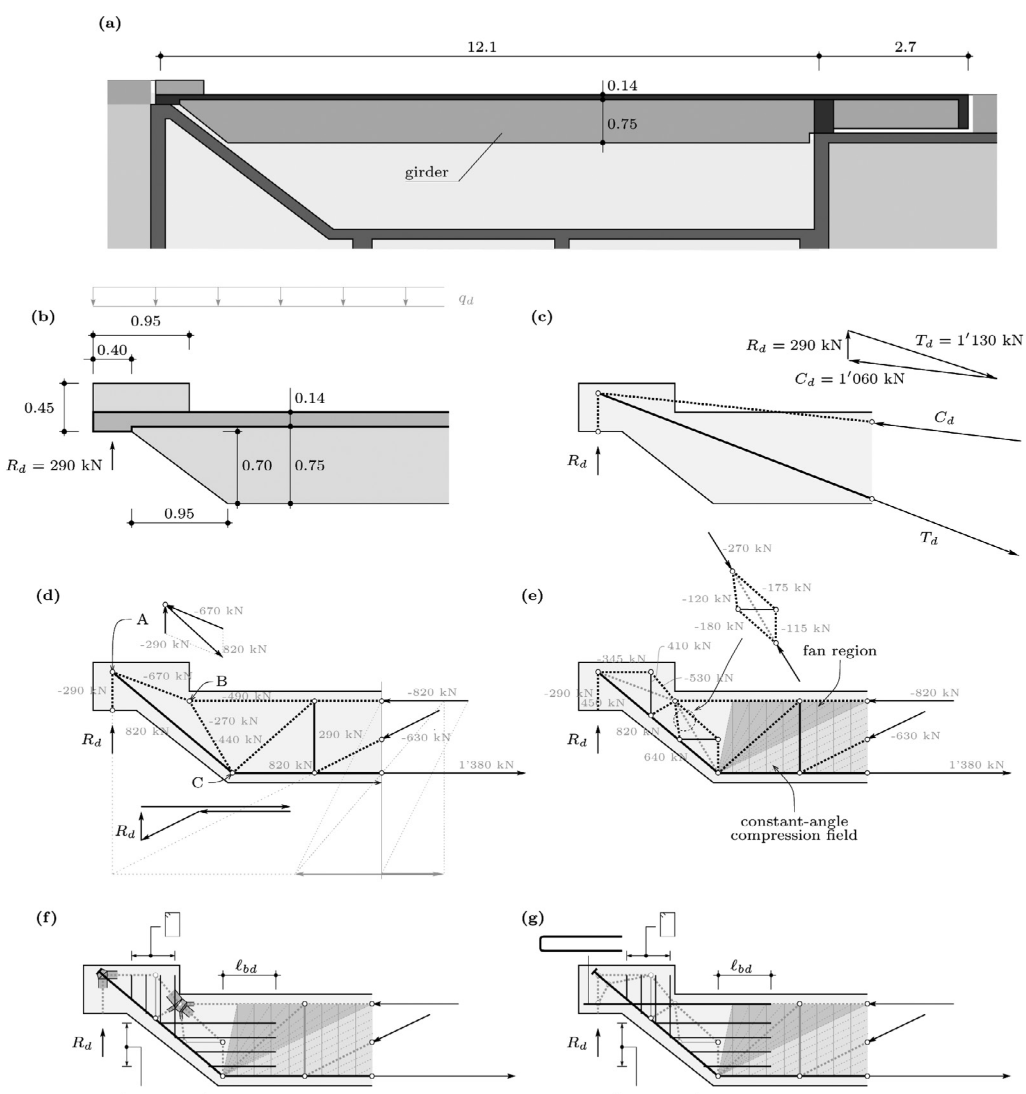

Figure 9. Example of application: (a) element investigated; (b) critical detail; (c) analysis based on thrust line; (d) strut-and-tie model (with lines for analysis with graphic statics shown in light grey); (e) refined strut-and-tie model; (f) stress field verifications and reinforcement layout; and (g) enhanced detailing.

oping between the bent of the reinforcement and the convex corner of the compression face (point B in Figure 9d). As it can be noted, the value of factor ν becomes very low in the region where the compression field is rather parallel to the tie (refer to dark-shaded area in Figure 10c). The member thus fails with a severely reduced concrete strength before yielding of any reinforcement (failure attained at R_{Rd} = 110 kN, lower than the applied action R_d = 290 kN).

A suitable response is on the contrary confirmed when the reinforcement is arranged according to the strut-and-tie model of Figure 9e. The value of factor ν is consistent with the one proposed by the codes (refer to Figure 10g) and failure occurs by yielding of the flexural reinforcement. It can be noted that the yielded zone of the inclined reinforcement (indicated in brown in Figure 10e) develops at the same location as the critical zone according to the refined strut-and-tie model (Figure 9e). In this case, with the reinforcement designed according to the lower-bound solution of Figure 9f, it results a member resistance equal to R_{Rd} = 351 kN. The over-strength with respect to the design load is mainly justified by the activation of the horizontal stirrups as flexural reinforcement in the critical region and by rounding of the required diameters of the flexural bars. It can also be noted the important role of the horizontal reinforcement near to the loading plate, which deviates the load introduction (local yielding in Figure 10e). Such reinforcement can, in fact, be increased to avoid local cracking issues, as shown in Figure 9g. Finally, it shall be considered that verification of the cracking state or other serviceability limit states might be governing, which can be performed according to specific models (*fib* 2021).

8.
CONCLUSIONS

The current draft for the revision of Eurocode 2 (FprEN1992-1-1:2022) maintains the strut-and-tie method as a basic tool for the design of discontinuity regions of concrete structures. Its scope has been enlarged by introducing the stress field method for verification of the compression fields and nodal regions and the full consistency of the two approaches is highlighted in the new standard. As a result, the designer has a consistent tool to design both discontinuity regions (where the assumption that plane sections remain plane does not hold) and beam regions (where deformed sections can be assumed to remain plane). Also, the same method can be consistently applied for the design of membrane elements. The provisioned rules are in addition simple to apply and have clear physical meaning, enhancing the ease-of-use of the code and the understanding of the code by engineers.

The draft for the new Eurocode 2 also encourages the use of refined analyses based on the stress field method. Such analyses consider the compatibility of deformations and allow more accurate estimates of the strength reduction factor accounting for the state of concrete cracking. These analyses are particularly useful for the assessment of existing structures, where the different load-carrying actions can be considered in an explicit manner.

It is the belief of the authors that the changes introduced in the code will address in a more comprehensive manner the challenges of structural engineers in the years to come, providing them with a sound tool for understanding, designing and assessing structural concrete.

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Notation

A_p	:	cross section area of prestressing
A_s	:	cross section area of reinforcement
C_d	:	design value of compression force
E_s	:	modulus of elasticity of reinforcement
F_{Rd}	:	design value of resistance in tension
F_{td}	:	design value of tension force
R_d	:	design value of reaction
T_d	:	design value of tension force
f_{cd}	:	design value of uniaxial compressive resistance of concrete
f_{ck}	:	characteristic value of uniaxial compressive resistance of concrete
f_{cp}	:	plastic strength of concrete in compression
f_{pd}	:	design value of the yield strength of the prestressing
f_y	:	yield strength of reinforcement
f_{yd}	:	design value of the yield strength of reinforcement
l_{bd}	:	design value of anchorage length
$l_{bd,red}$	:	design value of anchorage length (reduce by transverse pressure)
ε_1	:	principal tensile strain
ε_2	:	principal compressive strain
ε_c	:	strain in compression chord
ε_{cr}	:	crushing strain of concrete
ε_t	:	strain in tension chord
ε_x	:	strain in the x direction
γ	:	shear strain
γ_C	:	partial safety factor of concrete
η_{fc}	:	brittleness factor of concrete
ν	:	compression softening efficiency factor for concrete cracking
σ_{cd}	:	design value of the stress in the concrete
σ_{c1}	:	principal tensile stress of concrete
σ_{c2}	:	principal compressive stress of concrete
θ_{cs}	:	inclination of compression field

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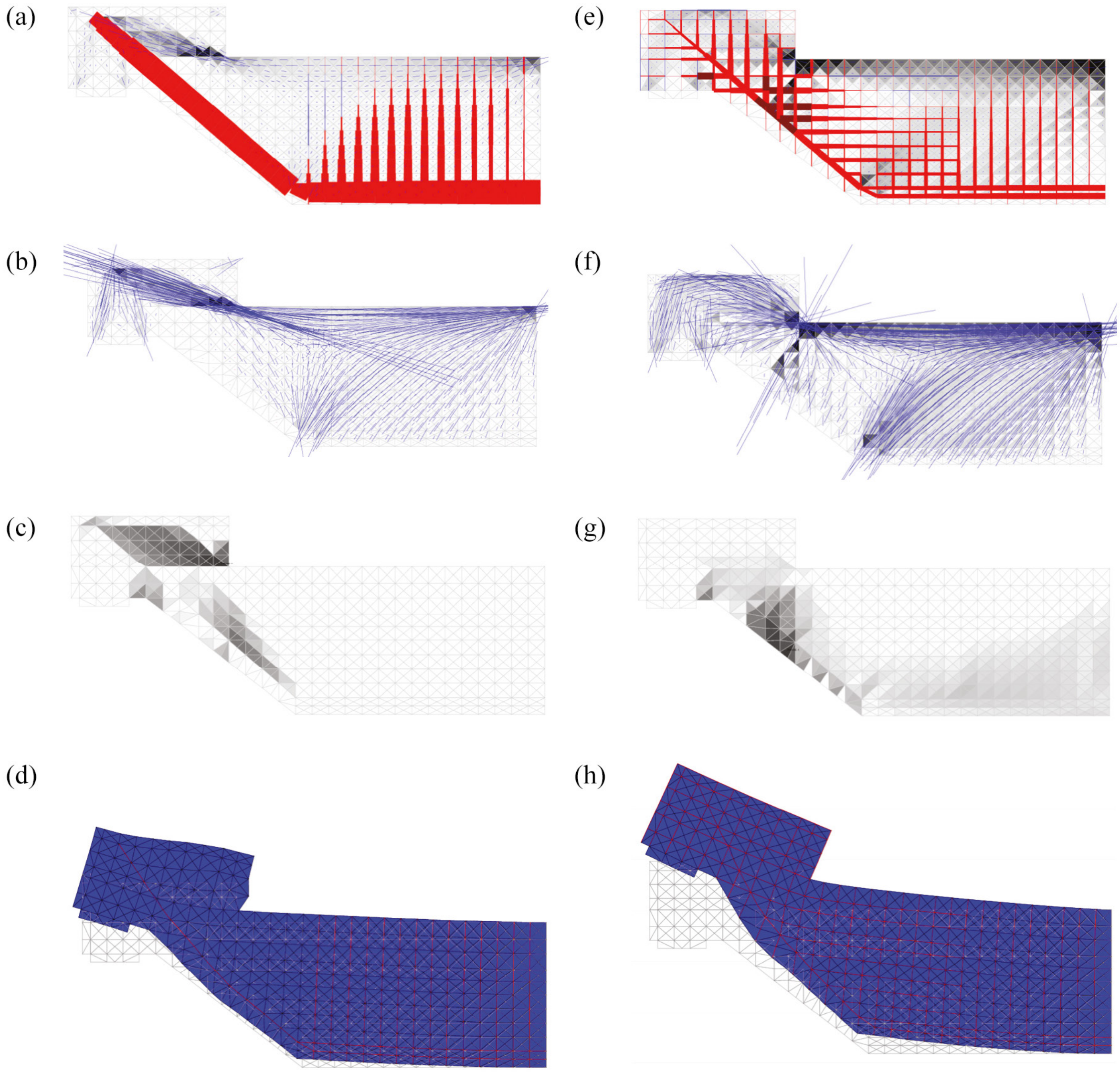

Figure 10. Example of application, use of EPSF: (left column) case with only inclined reinforcement; and (right column) case with additional horizontal and vertical reinforcement. Results at failure: (a,e) relative stresses at the reinforcement and concrete (red refers to tension and blue to compression; for the reinforcement, brown means yielding and for the concrete black means crushing); (b,f) detail of concrete stresses; (c,g) value of ν factor (black means $\nu = 0$, white $\nu = 1$); and (d.h) deformed shapes at failure.

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