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Performance-based slenderness limits for deformations and reinforcement stresses control in reinforced concrete beams

Límites de esbeltez basados en prestaciones para vigas de hormigón armado para el control de deformaciones y el control de tensiones en la armadura

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ABSTRACT

Due to the complex deformational behavior of cracked RC members, an effective way to ensure the fulfilment of the SLS is to limit the slenderness ratio *l/d* of the element. In this study, the deformation slenderness limit concept is generalized to incorporate crack width limitations. The proposed slenderness limits are compared with those derived from non-linear time-dependent analysis and also with those obtained using the EC2 method of deflections interpolation. Very good approximation and low scatter has been obtained showing that the proposed slenderness limits are a useful tool for performance-based design of RC structures.

KEYWORDS: Slenderness limits; deflection; crack width; SLS; performance-based design.

RESUMEN

Debido al complejo comportamiento en servicio de los elementos fisurados de hormigón armado, una forma efectiva de garantizar el cumplimiento de la verificación del estado límite de deformaciones es limitar la relación de esbeltez *l/d* del elemento. En este estudio, el concepto de esbeltez límite se generaliza para incorporar las limitaciones de abertura máxima de fisura. Los límites de esbeltez propuestos se comparan con los derivados del análisis no lineal dependiente del tiempo y también con los obtenidos utilizando el método de interpolación de flechas del Eurocódigo 2. Se ha obtenido una buena aproximación con una baja dispersión, lo que demuestra que los límites de esbeltez propuestos son una herramienta útil para el diseño basado en prestaciones de estructuras de hormigón armado.

PALABRAS CLAVE: límites de esbeltez; flecha; ancho de fisura; ELS; diseño basado en prestaciones.

1. INTRODUCTION

Excessive deformations may cause damage to non-structural elements, as well as problems related to aesthetics or functionality on Reinforced Concrete (RC) structures. To avoid excessive deflections that affect the serviceability performance of the structural members, their allowable design value is limited to a fraction of their span *l*. For instance, a limit of *l*/250 is indicated in the Eurocode 2 [1] for the deflection due to quasi-permanent loads. Likewise, a limit of *l*/500 is applicable for the increment of deflection after construction of partitions or other elements susceptible to be damaged.

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Actual deflections are difficult to predict, due to complex phenomena such as cracking, creep and shrinkage of concrete, and to the uncertainty associated to some governing parameters such as the concrete tensile strength. Furthermore, long-term deflections are influenced by environmental conditions, element dimensions, concrete properties, reinforcement ratios, construction sequence, value and duration of sustained loading and age at loading, among others. In this context, simplified and conservative methods have been adopted by the codes of practice and recommendations, such as the Eurocode EC2 [1], fib Model Code for Concrete Structures 2010 [2], ACI 318 [3], and others. Even so, there is an extensive literature about discussion, improvement, or further simplification of such simplified methods (Gilbert [4], Bischoff and Scalon [5], Mari *et al*. [6]

One of the most practical and effective ways to control excessive deflections is to provide the element with sufficient stiffness, which can be achieved by limiting the slenderness ratio, *l/d*, of the element. Furthermore, a proper selection of *l/d* may help in providing an adequate sizing of the cross section from the first steps of the design process thus contributing to its simplification. Different proposals and studies about limit slenderness ratios to avoid excessive deflections have been previously carried out. Among them, the most relevant are those carried out by Rangan [7] Gilbert [8], Scanlon and Choi [9], Lee and Scanlon [10], Bischoff and Scanlon [11], Bischoff [12], Pérez Caldentey *et al*. [13] and Gardner [14].

Control of cracking is another important aspect related to serviceability behavior of RC structures. Different parameters may influence crack width, but it is widely accepted that it is directly related to the tensile reinforcement strain (EC2 [1], MC2010 [2], Balazs and Borosnyoi [15], Gergely and Lutz [16], Frosch [17]). Strains (or stresses) in the tensile reinforcement can be calculated from the flexural moment distribution and sectional mechanical properties, and slenderness limits (as it is seen in the paper) related to a maximum stress in the reinforcement can be obtained. As a consequence, limitations of deflections may be related to the limitations of the cracks width required for aesthetic and durability reasons. Therefore, it can be said that it may be possible to find a domain of solutions in terms of *l/d*, reinforcement ratio and reinforcement stress or strain, which allow the simultaneous fulfilment of the SLS and the ULS of flexure. Barris *et al*. [18] studied the application of EC2 [1] formulation on SLS to Fiber Reinforced Polymer (FRP) RC flexural members, obtaining a formulation to obtain the slenderness limits that accomplish with the deflection limitation, maximum crack width and stresses in materials.

From the analysis of the existing literature, it is seen that there is not a unique accepted model to estimate the l/d ratio. Furthermore, the simultaneous fulfilment of a limit of stress intended for control of cracking is not taken into consideration. In the present study, the slenderness limit concept for deflection control is generalized to incorporate the crack width limitations in the framework of structural performance-based design.

2. SLENDERNESS RATIO ASSOCIATED TO DEFLECTION LIMITS

2.1 General

Consider a beam subjected to a dead load (g) and live load (q), uniformly distributed along the span length, so that the total load is $p = g + q$. Being ψ_2 the factor for the quasi-permanent load combination, the ratio between the quasi-permanent load and the total load, k_g, is defined as:

$$k_g = \frac{g+\psi_2 q}{g+q} \tag{1}$$

The long-term deflection (including the instantaneous and time-dependent deflections) produced by the quasi-permanent load combination must be limited to a fraction of the span length ($a_{qp} < l/C$)

$$a_{qp} = k_b \frac{k_g p l^4 k_t}{E_c I_{eff}} \leq \frac{l}{C} \tag{2}$$

where p is the total characteristic load ($g + q$); $k_g p$ is the quasi-permanent load; k_t is a factor that relates the time-dependent to the instantaneous deflection due to quasi-permanent loads; k_b is a factor to account for the support conditions (i.e. k_b=5/384 for simply supported members); l is the span length; C is a constant that indicates the fraction of the length for limitation of deflections (i.e., C=250 for the long-term deflection under the quasi-permanent load combination); I_{eff} is the effective moment of inertia, which takes into account concrete cracking and tension stiffening; and E_c is the modulus of elasticity of concrete.

In the next sections, each term of Eq. (2) will be derived and a simplified expression for the deflection slenderness limit will be obtained.

2.2 Effective moment of inertia Ieff and cracking factor kr

In the present study, it is considered that the members are cracked under the quasi-permanent load combination, assuming that in a certain moment, they could have been subject to the characteristic load. It is also taken into account that there are parts not cracked in the elements and that the concrete surrounding the reinforcement, placed between cracks contributes to the stiffness of the cracked regions (tension stiffening). Therefore, the following effective moment of inertia, I_{eff}, for computing deflections can be derived from the bilinear interpolation method for calculation of instantaneous deflections, provided by the MC2010 [2]:

$$I_{eff} = \frac{I_I I_{II}}{I_I \zeta + I_{II}(1-\zeta)} \tag{3}$$

where I_I and I_{II} are, respectively, the moments of inertia of the uncracked and the fully cracked sections and ζ is an interpolation coefficient, which depends on the type of load and level of cracking, given by:

$$\zeta = 1 - \beta \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2 \tag{4}$$

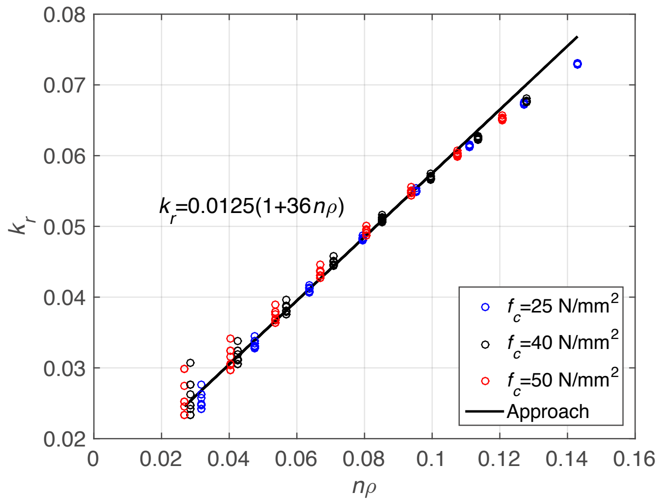


Figure 1. Cracking factor to obtain the effective inertia of a cracked section.

where β is a coefficient accounting for the type of loading (β = 0.5 for repeated or sustained loads); σ_s is the maximum attained stress in the tension reinforcement calculated on the basis of a cracked section under the load considered; and σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions that cause first cracking.

The un-cracked and fully cracked moments of inertia for a rectangular section of width b, effective depth d and total depth h can be obtained, neglecting the contribution of the compression reinforcement, by using the following equations:

$$I_I \cong I_g = \frac{bh^3}{12} \tag{5}$$

$$I_{II} = bh^3\, n\rho\,(1-\frac{x}{d})\,(1-\frac{x}{3d}) \tag{6}$$

where: $\rho=A_s/(bd)$ is the tensile reinforcement ratio; $n=E_s/E_c$ is the modular ratio between reinforcement and concrete; x is the neutral axis depth of the fully cracked section neglecting the compression reinforcement:

$$\frac{x}{d} = n\rho\left(1+\sqrt{1+\frac{2}{n\rho}}\right) \cong 0.75(n\rho)^{1/3} \tag{7}$$

Assuming an effective depth d = 0.9h, the value of I_{eff} for a cracked rectangular section, given by Eq. (3), can be well fitted by a straight line, see Figure 1, where dimensionless parameter $k_r = I_{eff}/bd^3$ is plotted as a function of the homogenized tensile reinforcement ratio $n\rho$, for reinforcement stresses at service of σ_s = 175 N/mm², σ_s = 225 N/mm² and σ_s = 275 N/mm². It can be observed that depends on $n\rho$ and is practically not influenced by the reinforcement stress level.

Therefore, the effective moment of inertia can be expressed as:

$$I_{eff} = k_r\, bd^3 = 0.0125\ (1+36n\rho)bd^3 \tag{8}$$

where k_r is the "cracking factor" that takes into account the tensile reinforcement ratio and the tension stiffening effect, given by Eq. (9)

$$k_r = 0.0125\ (1+36n\rho) \tag{9}$$

2.3 *Time-dependent deflections factor kt*

In order to obtain the increment of deflections due to creep and shrinkage, a time-dependent analysis of a cracked section subjected to a sustained load must be done. Due to the constraint produced by the steel to the increment of concrete strains along the time, a relaxation of the maximum compressive stress in concrete and an increment of the neutral axis depth and of the stresses in the compressive reinforcement take place. Furthermore, according to experimental observations, the strain at the tensile reinforcement is almost constant along the time, so the section can be assumed to rotate around the reinforcement, see figure 2 (Clarke *et al* [19], Murcia [20]. Such fact allows a considerable simplification of the time-dependent sectional analysis, with small errors.

Adopting the above assumption, Marí *et al* [21] derived factor k_t relating time dependent and instantaneous deflections, which is given by Eq. (10):

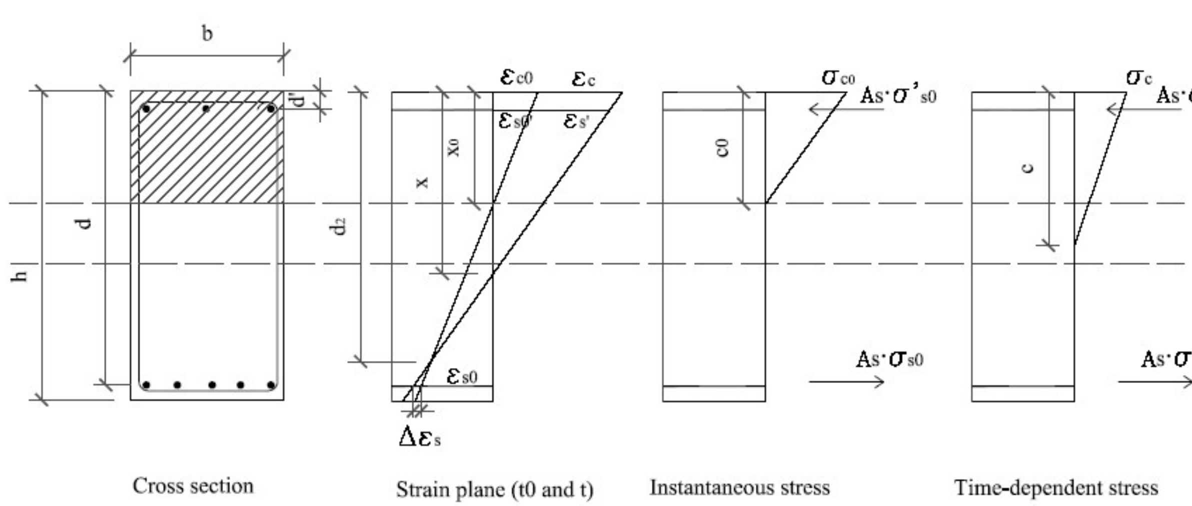


Figure 2. Time-dependent increment of stresses and strains in a RC cracked section.

$$k_t = 1 + \frac{0.24\varphi + 1000\varepsilon_{sh}}{1+12\, n\rho'} \quad (10)$$

where ϕ is the creep coefficient at time $t \geq t_0$, ε_{sh} is the shrinkage strain, and $\rho'=A_{s'}/b_d$ is the compression reinforcement ratio.

2.4 Slenderness associated to deflection limitation

Substituting Eq. (8) into Eq. (2), and after some arrangements, the following expression for the deflection slenderness limit, *l/d*, is derived:

$$\frac{l}{d} \leq \sqrt[3]{\frac{E_c\, k_r}{Ck_b\, k_g\, k_t \frac{p}{b}}} \quad (11)$$

where *p* is the characteristic uniformly distributed load; b is the beam width and *p/b* is the characteristic load applied per unit surface. Analyzing Eq. (11), some conclusions can be drawn: 1) the slenderness ratio *l*/d is lower for beams than for slabs because *p/b* is higher in the case of beams; 2) the higher the tensile and the compressive reinforcement ratios, the higher *l/d*, for the same load *p/b*, since k_r monotonically increases with ρ and k_t decreases when ρ' increases; 3) the higher the support constraints, the higher *l/d* (i.e. for continuous beams or frames, coefficient k_b is lower than for simply supported beams); 4) the higher the values of creep coefficient and shrinkage strain, the higher is k_t, and the lower is *l/d*; 5) the higher the concrete compressive strength, the higher *l/d* since, even though n and, consequently k_r, is lower, E_c is higher and k_t is lower.

For a member with given dimensions, materials and reinforcement ratio (i.e. designed to resist at least the design loads at ULS of flexure), Eq. (11) may be used to check whether it is necessary or not to calculate deflections for the verification of its corresponding limit state. Alternatively, Eq. (11) can be used to obtain the reinforcement amount necessary to satisfy the deformation limit state, solving it for k_r, which is directly related to $n\rho$ (see Eq. 9).

3.
SLENDERNESS ASSOCIATED SIMULTANEOUSLY TO DEFLECTION AND REINFORCEMENT STRESS LIMITATIONS

In order to satisfy the serviceability limit state of cracking, the crack width needs to be limited. The crack width depends on many factors associated to concrete, steel and bond properties, the acting bending moment, the reinforcement ratio and the bars diameter, among others. In particular, the reinforcement stress is a major factor influencing the crack width, so the computation of the average crack width can be avoided if certain relations between the reinforcement stress, the diameter and the spacing of the bars are satisfied, as stated by Eurocode 2 [1] and MC2010 [2]. For this reason, in this work, slenderness associated to a maximum allowable reinforcement stress under the quasi permanent load combination, $\sigma_{s,max}$, will be derived, as a way of limiting the crack width.

The stress in the tension reinforcement, σ_s, in a fully cracked section of rectangular shape or T-shape (when $x<h_f$), subjected to a bending moment M_{qp} produced by the quasi-permanent load combination, can be formulated as:

$$\sigma_s = \frac{M_{qp}}{zA_s} \cong \frac{k_g M}{0.9dA_s} = \frac{k_g k_m p l^2}{0.9\rho d^2} \quad (12)$$

where σ_{smax} is the limiting reinforcement stress to avoid excessive crack width; k_m is a factor relating the characteristic bending moment, *M*, with the characteristic load p and support conditions ($M = k_m\, p\, l^2$). The lever arm z = 0.9*d* has been adopted considering a neutral axis $x = 0.3d$, which corresponds to an average ratio ρ = 1.0 %, so that $z = d\text{-}x/3 \cong 0.9d$. Solving Eq. (12) for *l/d* and substituting it into Eq. (11) a slenderness associated to deflections and reinforcement stress limits is obtained:

$$\frac{l}{d} \leq \frac{E_c\, k_m\, k_r}{0.9\, C\rho\sigma_{s,max} k_b\, k_g\, k_t} \quad (13)$$

Figures 3a and 3b show the slenderness *l/d* associated to deflection, Eq. (11), and reinforcement stress limits, Eq. (12), for different steel reinforcement ratios (ρ) and surface loads (*p/b*), for simply supported beams (k_b = 5/384) and for external spans of continuous beams (k_b = 1/185), respectively, adopting f_{ck} = 30 N/mm², ϕ = 2.5, ε_{sh} = 0.0003, as concrete properties, deflection limitation C = 250 and a ratio of permanent to total loads k_g = 0.7.

A particular case of interest is that associated to the amount of reinforcement strictly necessary for flexural strength (which is the basis for the adjustment of EC2 [1] and MC2010 [2] slenderness limits). In this case, the stress in the reinforcement, under the quasi-permanent load combination, may be estimated as:

$$\sigma_{s,qp} = \frac{k_g\, f_{yd}}{\gamma_f} \quad (14)$$

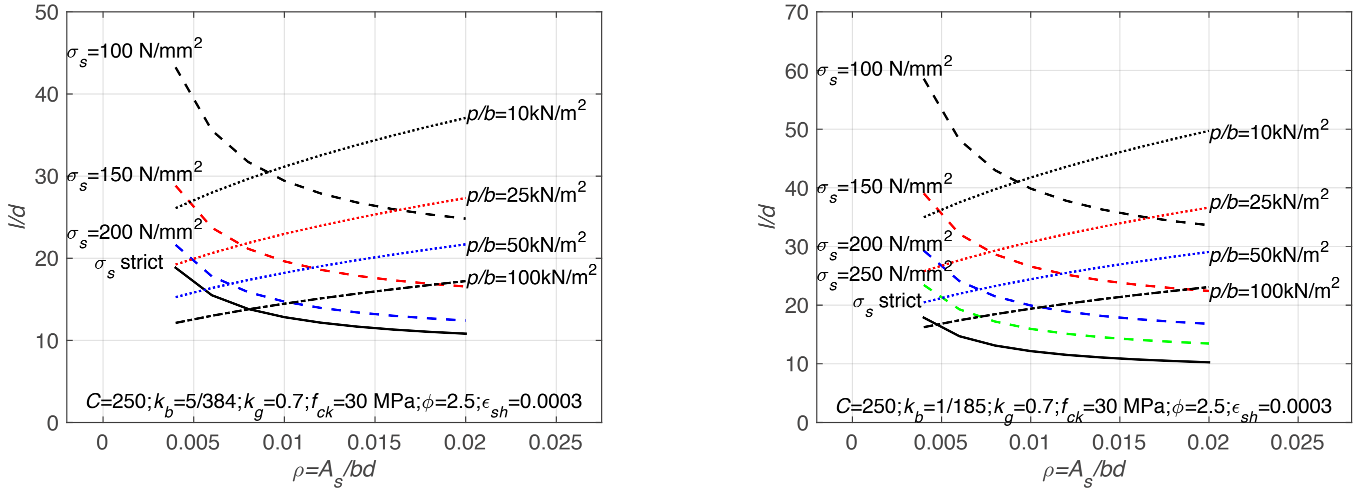
where γ_f is the average loads factor, which can be adopted as 1.4 for usual ratios of permanent to live load. The slenderness limit associated to such stress in the reinforcement is, then:

$$\frac{l}{d} \leq \frac{E_c\, \gamma_f\, k_m\, k_r}{0.9\, C\rho\, \gamma_f\, k_b\, k_g\, k_t} \quad (15)$$

which is plotted in figures 3a and 3b as "Strict" stress.

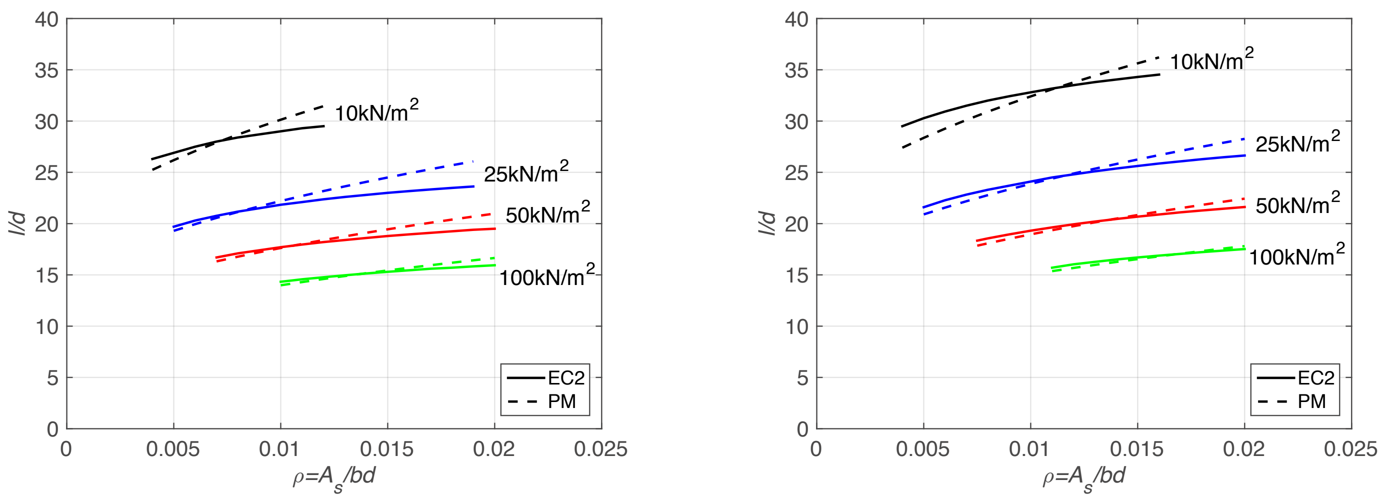
4.
COMPARISON OF THE PROPOSED SLENDERNESS LIMITS WITH THOSE OBTAINED COMPUTING DEFLECTIONS WITH THE EUROCODE 2

To analyze the capacity of the proposed method to obtain reasonable values of the slenderness limit, a comparison with results obtained using the EC2 [1], for the computation of deflections, is made in this section. According to previous sec-



a) b)

Figure 3. Deformation and stress limitation slenderness ratios, a) simply supported beams, b) external span continuous beams.



a) b)

Figure 4. Comparison between l/d values obtained using EC2 [1] and proposed method (PM) for constant load p/b=10, 25, 50 and 100 kN/m^2 a) f_{ck}=30 N/mm^2, b) f_{ck}=50 N/mm^2.

tions, the analysis has been done for values of l/d obtained for constant load, as well as for constant stress. The calculations have been done as explained in the following.

For the case of constant load, given a specific reinforcement ratio and sectional characteristics, a span length, l, is assumed, allowing obtaining long-term deflections due to quasi-permanent load from an effective moment of inertia calculated on the basis of interpolation between uncracked and fully cracked sections [1] [2]. The level of cracking for obtaining the effective moment of inertia is calculated by using the characteristic load. Trying different values of the span length, the slenderness is obtained dividing l by d, when the deflection is $l/250$.

A similar procedure has been used for the case of constant stress due to quasi-permanent loads. For a given reinforcement ratio, and a value of the stress in the tensile reinforcement, the service flexural moment for the critical section can be obtained. Again values for l are tried and the slenderness limit is obtained when the deflection is $l/250$.

This global procedure is not different from that used in other works [13] for obtaining the l/d value corresponding to the maximum bending moment associated to a given reinforcement ratio (strict value). However, here the values are obtained also for lower loads than those corresponding to the flexural capacity of the section, which is usually the case in practice.

Figure 4 shows the comparison for values of p/b of 10, 25, 50 and 100 kN/m^2, assuming f_{yk} = 500 N/mm^2, k_g=0.7, $g/(g+q)$=0.6, γ_f=1.41, ϕ = 2.5, and ε_{sh}=0.00035. Two representative characteristic concrete strengths, 30 and 50 N/mm^2, have been used in the analysis (figures 4a and 4b respectively) even though only a slight increment is observed with the concrete strength. An increase of l/d is seen for an increase of reinforcement ratio with constant load. A logical reduction in l/d is showed for increasing loads.

The proposed method (PM in figures 4a and 4b) follows reasonably well the values obtained with a much more com-

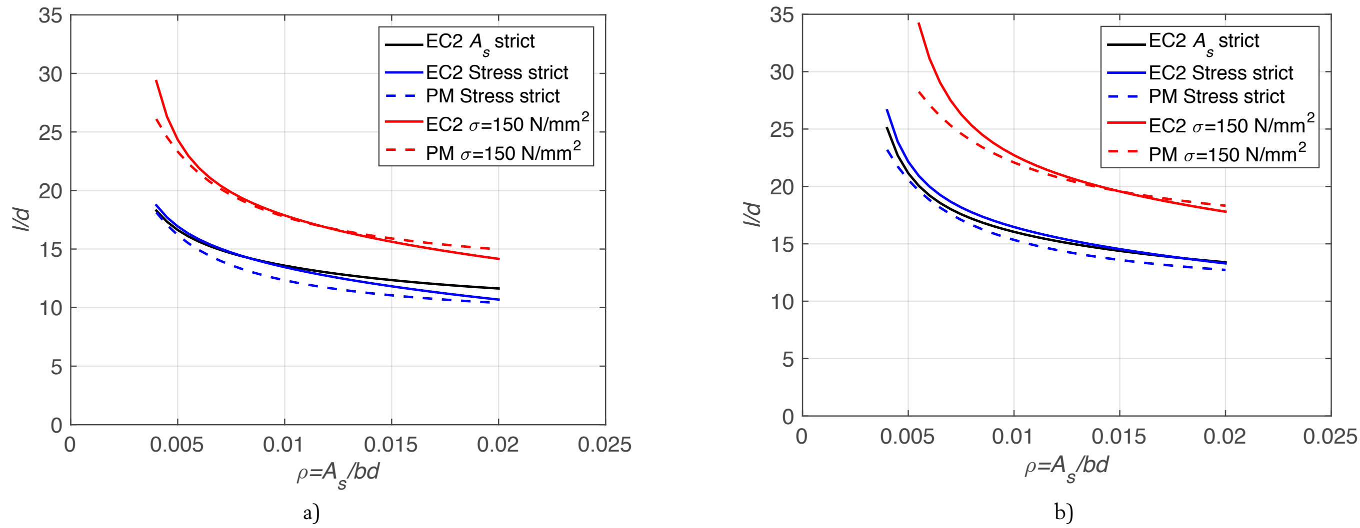


Figure 5. Comparison between l/d values obtained using EC2 [1] and proposed method (PM) for constant stress due to quasi-permanent load a) fck=30 N/mm², b) fck=50 N/mm².

plex model, such as that from EC2 [1]. Statistical values (average, maximum, minimum and coefficient of variation) of the ratio between slenderness limits obtained with the proposed method and that from EC2 [1] are shown in table 1. It is seen that average values are quite close to the unity. Maximum differences are obtained for the lowest load level, and as the load increases the curves are practically identical.

TABLE 1.
Statistical values of the ratio between *l/d* from proposed method and EC2 [1], for constant *p/b*

	f_{ck} = 30 N/mm²				f_{ck} = 50 N/mm²			
p_k	Avg.	Max.	Min.	COV	Avg.	Max.	Min.	COV
10	1.03	1.12	0.96	0.049	0.99	1.08	0.91	0.051
25	1.02	1.08	0.97	0.034	1.00	1.06	0.94	0.036
50	1.00	1.05	0.97	0.027	0.99	1.03	0.96	0.025
100	1.00	1.03	0.97	0.019	0.98	1.00	0.97	0.010

Figure 5 shows the comparison for values of constant stress of 150 N/mm² due to quasi-permanent loads, as well as those obtained for the maximum permissible stress under serviceability conditions, corresponding to that of the steel yielding strength for ultimate limit state ($f_{yd} = f_{yk}/\gamma_s$ = 500/1.15 = 435 N/mm²), which is named in the figures as "σ_{strict}". As indicated previously, in these circumstances the quasi-permanent stress would be $f_y d\ k_g/\gamma_f$ = 435·0.7/1.41 = 216 N/mm².

For comparison purposes another curve called "EC2-As strict" is also presented. This curve is obtained using the procedure that was followed for obtaining the EC2 [1] slenderness ratios. It represents the values corresponding to the service moment obtained from the ultimate bending moment corresponding to a given reinforcement ratio. The difference with the "σ_{strict}" curve is that in this case the maximum bending moment is calculated under ULS, while in the previous case is calculated from serviceability conditions (limiting the quasi-permanent service stress); the difference in the lever arms in the calculation gives the slightly different curves. Figures 5a and 5b show similar trends. In this case some more difference than for the case of constant load can be seen at low reinforcement ratios for the two characteristics strength used. As seen in subsection 2.5 an increase in reinforcement ratio causes a reduction in *l/d*, since keeping the stress constant leads to a higher flexural moment to be sustained.

Statistical values of the ratios between both methods are reported in table 2, showing that the proposed method provides acceptable values for design. The maximum differences are obtained for the lowest reinforcement ratios, probably due to the fact that for low reinforcement ratios the moment at service is not much higher than the cracking moment and, therefore, tension stiffening is relevant. Furthermore, the assumption made about constant strain at the tensile reinforcement along the time may deviate from the actual value for low reinforcement ratios. In any case, the errors are of acceptable magnitude and in the safe side.

TABLE 2.
Statistical values of the ratio for constant stress between *l/d* from proposed method and EC2 [1], for constant *p/b*

	f_{ck} = 30 N/mm²				f_{ck} = 50 N/mm²			
Stress	Avg.	Max.	Min.	COV	Avg.	Max.	Min.	COV
150 N/mm²	0.98	1.05	0.86	0.040	0.95	1.02	0.80	0.054
Strict	0.94	0.98	0.92	0.021	0.91	0.96	0.84	0.026

5. CONCLUSIONS

The following conclusions can be drawn from the work done: Slenderness limits (*l/d*) for RC beams, associated to given limitations of deflections under the quasi-permanent load combination and limitations of stresses in the reinforcing steel, for crack control, have been derived. The derived equations are simple to use in design, either to know the minimum beam depth or the minimum reinforcement ratio necessary to avoid calculation of deflections or excessive crack width.

Very simple expressions have been derived for the effective inertia accounting for tension stiffening, and a time factor *kt*, which allows obtaining the long-term deflections due to concrete creep and shrinkage, from the instantaneous ones.

A comparative study has been made between the proposed slenderness limits with those obtained by calculating the long-term deflections by means of Eurocode 2 [1], studying the influence of reinforcement ratio, concrete strength, load and stress levels. Very good agreement has been obtained for the most common cases, although differences up to 20 % (on the side of safety) have been found for low reinforcement ratios and low levels of stress and load.

The way in which the slenderness limits have been obtained, based on the mechanics of reinforced concrete and on an experimentally verified allows its application to a large variety of structural situations (i.e. support constraints, environmental conditions, materials properties, quasi-permanent load factors, etc).

Acknowledgements

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References

[1] EN 1992-1-1 (2004). (English) *Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings*. [Authority: The European Union Per Regulation 305/2011, Directive 98/34/EC, Directive 2004/18/EC]. Brusells: CEN.

[2] Federation International du Béton (fib) (2013). *Model Code for Concrete Structures 2010*. Berlin: Verlag Ernst & Sohn.

[3] ACI-Committee-318 American Concrete Institute. ACI318-11. (2011). *Building Code Requirements of Structural Concrete and Commentary*.

[4] Gilbert, R.I. (1999). Deflection calculation for reinforced concrete structures- why we sometimes get it wrong. *ACI Struct J*, 96(6), 1027-1032. https://doi.org/10.14359/779.

[5] Bischoff, P.H. & Scanlon, A. (2007). Effective moment of inertia for calculating deflections of concrete members containing steel reinforcement and fiber-reinforced polymer reinforcement. *ACI Struct J*, 104(1), 68–75. https://doi.org/10.14359/18434

[6] Marí, A., Bairán, J. & Duarte, N. (2010). Long-term deflections in cracked reinforced concrete flexural members. *Eng Struct*, 32(3), 829-842. https://doi.org/10.1016/j.engstruct.2009.12.009.

[7] Rangan, V. (1982). Control of beam deflections by allowable span-depth ratios. *ACI J*, 79, 372-377.

[8] Gilbert, R.I. (1985). Deflection control of slabs using allowable span to depth ratios. *ACI J*, 82(1), 67–72.

[9] Scanlon, A. & Choi, B.S. (1999). Evaluation of ACI 318 minimum thickness requirements for one-way slabs. *ACI Struct J*, 96(4), 616-621. https://doi.org/10.14359/699.

[10] Lee, Y.H. & Scanlon A. (2010). Comparison of one- And two-way slab minimum thickness provisions in building codes and standards. *ACI Struct J*, 107(2), 157-163. https://doi.org/10.14359/51663531.

[11] Bischoff, P.H. & Scanlon, A. (2009). Span-depth ratios for one-way members based on ACI 318 deflection limits. *ACI Struct J*, 106(7), 617-626. https://doi.org/10.14359/51663102.

[12] Bischoff, P.H. (2005). Reevaluation of Deflection Prediction for Concrete Beams Reinforced with Steel and Fiber Reinforced Polymer Bars. *Struct Eng*, 131 (5), 752-767. https://doi.org/10.1061/(ASCE)0733-9445(2005)131:5(752).

[13] Caldentey, A.P., Cembranos, J.M. & Corres Peiretti, H. (2017). Slenderness limits for deflection control: A new formulation for flexural reinforced concrete elements. *Struct Concr*, 18(1), 118-127. https://doi.org/10.1002/suco.201600062.

[14] Gardner, N.J. (2011). Span/thickness limits for deflection control. *ACI Struct J*, 108(4), 453-460. https://doi.org/10.14359/51682985.

[15] Balazs, G.L. & Borosnyoi, A. (2005). Models for Flexural Cracking in Concrete: The State of the Art. *Struct Concr*, 6(2), 53-62. https://doi.org/10.1680/stco.2005.6.2.53.

[16] Gergely, P. & Lutz, L.A. (1968). Maximum crack width in reinforced concrete flexural members. Causes, Mech Control Crack Concr SP20. American Concrete Inst Journal & Proceedings, 87-117.

[17] Frosch, R.J. (1999). Another look at cracking and crack control in reinforced concrete. *ACI Struct J*, 96(3), 437-442. https://doi.org/10.14359/679

[18] Barris, C., Torres, L., Mias, C. & Vilanova, I. (2012). Design of FRP RC beams for serviceability requirements. *Journal of Civil Engineering and Management*, 18(6), 843-857. https://doi.org/10.3846/13923730.2012.720934.

[19] Clarke, G., Scholz, H. & Alexander, A. (1988). New method to predict the creep deflection of craked reinforced concrete flexural members. *ACI Mater J.*, 85(2), 95-101.

[20] Murcia, J. (1991). Approximate Time Dependent Analysis of Reinforced Concrete Sections. Proposal of a New Factor for the Calculation of Long Term Deflections (in Spanish). *Hormigón y Acero*, 42(181), 9–17. http://www.hormigonyacero.com/index.php/ache/article/view/697

[21] Marí, A., Torres, Ll., Oller, E. & Barris, C. (2019). Performance-based slenderness limits for deformations and crack control of reinforced concrete flexural members. *Eng Structures* 187, 267-279. https://doi.org/10.1016/j.engstruct.2019.02.045